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MATHEMATICAL SIMULATION OF FLUID FLOW IN HYDRAULIC DRIVES OF APPARATUSES

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Problems of numerical simulation of branched hydraulic systems are considered. Difference schemes in Riemann invariants are constructed for nonlinear equations of poorly compressed fluid that are prescribed on hydraulic networks. Results of numerical simulation of fluid flow in the hydraulic drive of a mechanism for lowering cargo are presented.

Problems of the study and calculation of forced fluid flows play an important role in the design and manufacture of hydraulic drives. Work of hydraulic drives is characterized by the high dynamic nature of the processes and can occur under conditions of considerable load fluctuations and pulsating feed of fluid. When the frequency of fluid feed pulsations coincides with the resonance frequencies of hydraulic lines, valves, and other apparatuses, fluid oscillations may be multiply amplified by the amplitude. The thus-arising resonance phenomena and vibrations deteriorate the operation characteristics of hydraulic drives and reduce their reliability. Therefore, problems of the dynamics of a hydraulic drive and its design parameters with allowance for transient processes lead to the necessity of thorough study of the processes occurring in it by the methods of mathematical simulation.

The specifics of the calculation of branched hydraulic systems (hydraulic networks) lies in the necessity of integrating nonstationary equations in partial derivatives the region of whose determination is associated with the graph [1, 2]. In [3, 4], computational algorithms are suggested for solving systems of interrelated equations of hydraulics determined on a finite coupled graph under the simplest conjugation conditions assigned in the nodes of the graph. On the basis of implicit difference schemes in the Riemann invariants, we constructed computational algorithms for nonlinear equations of hydraulics prescribed on the hydraulic network. In this case, models of basic apparatuses of hydraulic drives were used as the conjugation conditions in the nodes of the network [5]. The efficiency of the suggested algorithms is demonstrated by the example of numerical simulation of fluid flow in the hydraulic drive of a mechanism for lowering cargo.

1. Formulation of the Problem. We consider a branched hydraulic system which can be presented as a coupled graph G. Structural parts of the hydraulic system (pipelines) constitute the edges of the graph and structural units (branches of pipelines, hydraulic cylinders, pumps, safety valves, etc.) serve as nodes. The nodes and edges of the graph are numbered independently of each other; on each edge we arbitrarily take a positive direction. We denote sets of the numbers of inner nodes, limiting nodes, and edges of the graph by $\eta = \{1, 2, ..., n\}$, $\eta_\Gamma = \{n, +1, ..., n + n_\Gamma\}$, and $\gamma = \{1, 2, ..., M\}$, respectively; $\gamma^+_{(q)}$ and $\gamma^-_{(q)}$ are the sets of numbers of the edges with the right and left end, respectively, adjacent to the qth node ($q \in \eta \cup \eta_\Gamma$).

Structural parts of the hydraulic system (edges of the graph G) are the elements with distributed parameters, and the processes in them are described by differential equations in partial derivatives. In the structural units (nodes of the graph G), unsteady flow of fluid can be described by either algebraic or ordinary differential equations relating fluid parameters in the unit and time.

To describe unsteady processes in a hydraulic drive the following mathematical models of the main elements of the hydraulic drive are used:

(a) Pipeline. In the region $(x, t) \in \bigcup_{k \in \gamma} \Omega_k \times (0, T)$, where $\overline{\Omega}_k = \{x_k: 0 \le x_k \le L_k\}$ is the edge of the graph G with the number k, fluid flow is described by the following system of one-dimensional hydrodynamic equations:

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$$\frac{\partial v_k}{\partial t} + v_k \frac{\partial v_k}{\partial x_k} + \frac{1}{\rho}\frac{\partial p_k}{\partial x_k} + \frac{\lambda v_k \, |v_k|}{2d_k} = 0\,, \tag{1}$$

$$\varepsilon \frac{\partial p_k}{\partial t} + \frac{\partial v_k}{\partial x_k} + \varepsilon v_k \frac{\partial p_k}{\partial x_k} = 0\,,$$

$$\varepsilon = (\rho c_k^2)^{-1}\,, \quad c_k = \sqrt{E_{\mathrm{red}}/\rho}\,, \quad E_{\mathrm{red}} = E_{\mathrm{fl}}\,(1 + d_k\, E_{\mathrm{fl}}/(\delta_k\, E_{\mathrm{w}}))\,.$$

The adjoining conditions are set on the region boundaries

$$\begin{aligned} p_k^- &= (p_k + \zeta_k^- \rho \, |v_k| \, v_k/2)\,|_{x_k=0}\,, \quad \zeta_k^- \geq 0\,, \quad k \in \gamma_{(q_1)}^-\,; \\ p_k^+ &= (p_k - \zeta_k^+ \rho \, |v_k| \, v_k/2)\,|_{x_k=L_k}\,, \quad \zeta_k^+ \geq 0\,, \quad k \in \gamma_{(q_2)}^+\,, \end{aligned} \tag{2}$$

These conditions express pressure-head losses for overcoming local hydraulic drags ζ_k^- , ζ_k^+ at the inlet and outlet from the kth pipeline.

(b) Branching of pipelines. Balance equations of the form

$$\sum_{k \in \gamma_{(q)}^-} f_k v_k \,|_{x_k=0} - \sum_{k \in \gamma_{(q)}^+} f_k v_k \,|_{x_k=L_k} = 0\,. \tag{3}$$

are assigned in the corresponding node q of the hydraulic network.

(c) Hydraulic cylinder. The dynamics of the hydraulic cylinder is described by the equation of piston translational motion due to pressure forces, outer load, and friction forces

$$f_1 p_{(q)}^+ - f_2 p_{(q)}^- = m\,\frac{du}{dt} + bu + F_1\,(z) + F_2\,(t)\,, \quad u = \frac{dz}{dt}\,, \quad 0 \leq z \leq L_{\mathrm{cyl}}\,, \tag{4}$$

and by the equations of flow rates with allowance for fluid compression in cavities:

$$\frac{dp_{(q)}^+}{dt} = \frac{1}{k_1}\,(f_k v_k \,|_{x_k=L_k} - f_1 u)\,, \quad k \in \gamma_{(q)}^+\,, \quad \frac{dp_{(q)}^-}{dt} = \frac{1}{k_2}\,(f_2 u - f_l v_l \,|_{x_l=0})\,, \quad l \in \gamma_{(q)}^-\,. \tag{5}$$

Here and in what follows, for hydraulic elements whose mathematical models include pressure at the inlet $p_{(q)}^+$ and outlet from the element $p_{(q)}^-$, we take the element inlet to be connected with the outlet of the kth pipeline, and the element outlet with the inlet of lth pipeline: $p_{(q)}^+ = p_k^+$, $k \in \gamma_{(q)}^+$, $p_{(q)}^- = p_l^-$, $l \in \gamma_{(q)}^-$.

(d) Check and safety valves. With no regard for fluid compression in valve channels, the flow-rate equations have the form (3), the value of flow is determined by the flow rate–pressure drop characteristic of the valve $Q(\Delta P_{(q)})$:

$$f_k v_k \,|_{x_k=L_k} = f_l v_l \,|_{x_l=0} = Q\,(\Delta p_{(q)})\,, \quad \Delta p_{(q)} = p_{(q)}^+ - p_{(q)}^-\,, \quad Q\,(\Delta p_{(q)}) = 0\,, \quad \Delta p_{(q)} \leq \Delta p_{\mathrm{val}}\,. \tag{6}$$

(e) Heat exchanger. Fluid flow rate through the heat exchanger is related with the pressure drop at the inlet and outlet by the relationship

$$p_{(q)}^+ - p_{(q)}^- = 8584 n_{\mathrm{h.ex}}\, l_{\mathrm{h.ex}}\, v_{\mathrm{h.ex}}^{1.42}\,, \quad f_k v_k \,|_{x_k=L_k} = f_l v_l \,|_{x_l=0} = 0.25\pi d_{\mathrm{h.ex}}^2 v_{\mathrm{h.ex}}\,, \tag{7}$$

where $n_{\mathrm{h.ex}}$ is the number of paths in the heat exchanger; $l_{\mathrm{h.ex}}$, $d_{\mathrm{h.ex}}$ are the path length and conditional passage of the heat exchanger.

(f) Pump. The pump is characterized by the feed of fluid $Q(t)$, where $Q(t)$ is a set function of time.

(g) Drainage. The boundary-value condition $p_{(q)} = p_{\text{atm}}$, $p_{\text{atm}} = 1.01325\cdot 10^5$ Pa, is set in the corresponding node of the hydraulic network.

Thus, the problem of mathematical simulation of fluid flow in the hydraulic system lies in integration for $t > 0$ of the systems of equations (1) with conjugation conditions of the form (2)-(7) and given initial data $v_k(x_k, t)\,|_{t=0} = v_k^0(x_k)$, $p_k(x_k, t)\,|_{t=0} = p_k^0(x_k)$.

2. Computational Algorithms. The system of equations (1) when $|v_k| < c_k$ is hyperbolic and can be presented in Riemann invariants $r_k = p_k + \rho c_k v_k$, $s_k = p_k - \rho c_k v_k$ in the form

$$\frac{\partial r_k}{\partial t} + (\varphi_k + c_k)\frac{\partial r_k}{\partial x_k} = -\psi_k(r_k - s_k)\,, \quad \frac{\partial s_k}{\partial t} + (\varphi_k - c_k)\frac{\partial s_k}{\partial x_k} = \psi_k(r_k - s_k)\,, \tag{8}$$

where $\varphi_k = (r_k - s_k)/(2\rho c_k)$, $\psi_k = \lambda|\varphi_k|/(4d_k)$. We introduce the node grid

$$\omega_{h\tau} = \omega_h \times \omega_\tau\,,$$

$$\omega_h = \left\{x_{k,i+1} = x_{k,i} + h_{k,i}\,,\ i = \overline{0, N_k - 1}\,,\ x_{k,0} = 0\,,\ x_{k,N_k} = L_k\right\},$$

$$\omega_\tau = \left\{t_j = j\tau\,,\ j = \overline{0, j_0}\,,\ j_0\tau = T\right\}.$$

We approximate by the difference scheme the system of equations (8):

$$\begin{aligned} &r_{kt} + (\hat{\varphi}_k + c_k)\,\hat{r}_{k\bar{x}} = -\hat{\psi}_k(\hat{r}_k - \hat{s}_k)\,,\quad i = 1, \dots, N_k\,, \\ &s_{kt} + (\hat{\varphi}_k - c_k)\,\hat{s}_{kx} = \hat{\psi}_k(\hat{r}_k - \hat{s}_k)\,,\quad i = 0, \dots, N_k - 1\,. \end{aligned} \tag{9}$$

Here $f_k = f^j_{k,i}$, $\hat{f}_k = f^{j+1}_{k,i}$, $f_{kt} = (\hat{f}_k - f_k)/\tau$, $f_{kx} = (f^j_{k,i+j} - f^j_{k,i})/h_{k,i}$. To realize (9) we use the iteration process

$$\begin{aligned} &\overset{m+1}{r}_{kt} + \left(\overset{m}{\varphi}_k + c_k\right)\overset{m+1}{r}_{k\bar{x}} = -\overset{m}{\psi}_k\left(\overset{m+1}{r}_k - \overset{m}{s}_k\right),\quad i = 1, \dots, N_k\,, \\ &\overset{m+1}{s}_{kt} + \left(\overset{m}{\varphi}_k - c_k\right)\overset{m+1}{s}_{kx} = \overset{m}{\psi}_k\left(\overset{m}{r}_k - \overset{m+1}{s}_k\right),\quad i = 0, \dots, N_k - 1\,. \end{aligned} \tag{10}$$

Expressions (10) are the "running calculation" scheme

$$\overset{m+1}{r}_{k,i} = \xi^+_{k,i}\,\overset{m+1}{r}_{k,i-1} + \upsilon^+_{k,i}\,,\quad i = 1, \dots, N_k\,,\quad \overset{m+1}{s}_{k,i} = \xi^-_{k,i}\,\overset{m+1}{s}_{k,i+1} + \upsilon^-_{k,i}\,,\quad i = N_k - 1, \dots, 0\,, \tag{11}$$

where

$$\xi^+_{k,i} = \frac{\vartheta^+_{k,i}}{1 + \vartheta^+_{k,i} + \tau\overset{m}{\psi}_{k,i}}\,,\quad \upsilon^+_{k,i} = \frac{r_{k,i} + \tau\overset{m}{\psi}_{k,i}\overset{m}{s}_{k,i}}{1 + \vartheta^+_{k,i} + \tau\overset{m}{\psi}_{k,i}}\,,\quad \vartheta^+_{k,i} = \frac{\tau}{h_{k,i-1}}\left(\overset{m}{\varphi}_{k,i} + c_k\right),$$

$$\xi^-_{k,i} = \frac{\vartheta^-_{k,i}}{1 + \vartheta^-_{k,i} + \tau\overset{m}{\psi}_{k,i}}\,,\quad \upsilon^-_{k,i} = \frac{s_{k,i} + \tau\overset{m}{\psi}_{k,i}\overset{m}{r}_{k,i}}{1 + \vartheta^-_{k,i} + \tau\overset{m}{\psi}_{k,i}}\,,\quad \vartheta^-_{k,i} = -\frac{\tau}{h_{k,i}}\left(\overset{m}{\varphi}_{k,i} - c_k\right).$$

To determine the invariants $\overset{m+1}{r}_{k,0}$, $\overset{m+1}{s}_{k,N_k}$ we obtain the relations

$$\overset{m+1}{r}_{k,N_k} = \xi^+_k\,\overset{m+1}{r}_{k,0} + \upsilon^+_k\,,\quad \overset{m+1}{s}_{k,0} = \xi^-_k\,\overset{m+1}{s}_{k,N_k} + \upsilon^-_k\,, \tag{12}$$

$$\xi_k^+ = \prod_{i=1}^{N_k} \xi_{k,i}^+ , \quad v_k^+ = \prod_{\alpha=1}^{N_k} v_{k,\alpha}^+ \prod_{i=\alpha+1}^{N_k} \xi_{k,i}^+ , \quad \xi_k^- = \prod_{i=0}^{N_k-1} \xi_{k,i}^- , \quad v_k^- = \sum_{\alpha=0}^{N_k-1} v_{k,\alpha}^- \prod_{i=0}^{\alpha-1} \xi_{k,i}^- .$$

Approximating the adjoining conditions (2) by $\omega_{h\tau}$ with allowance for (12), we obtain systems of linear equations relating the invariants $\overset{m+1}{r}_{k,0}$, $\overset{m+1}{s}_{k,N_k}$ and pressures $\overset{m+1}{p_k^-}$, $\overset{m+1}{p_k^+}$:

$$\begin{aligned} &(1+\theta_k^-)\, \overset{m+1}{r}_{k,0} + (1-\theta_k^-)\, \xi_k^- \, \overset{m+1}{s}_{k,N_k} - 2\, \overset{m+1}{p_k^-} = -(1-\theta_k^-)\, v_k^- , \\ &(1-\theta_k^+)\, \xi_k^+ \, \overset{m+1}{r}_{k,0} + (1+\theta_k^+)\, \overset{m+1}{s}_{k,N_k} - 2\, \overset{m+1}{p_k^+} = -(1-\theta_k^+)\, v_k^+ , \end{aligned} \tag{13}$$

$$\theta_k^- = 0.5\zeta_k^- \left| \overset{m}{v}_{k,0} \right| \Big/ c_k , \quad \theta_k^+ = 0.5\zeta_k^+ \left| \overset{m}{v}_{k,N_k} \right| \Big/ c_k .$$

Let us introduce into consideration the vector p = $(p_{(1)}, p_{(2)}, \dots , p_{(n)})'$ of nodal pressures in the inner nodes of the hydraulic network. We assume the following for the nodes that are branching points of pipelines:

$$p_{(q)} = p_{(q)}^+ , \quad k \in \gamma_{(q)}^+ , \quad p_{(q)} = p_k^- , \quad k \in \gamma_{(q)}^- , \tag{14}$$

and for the nodes characterized by pressures $p_{(q)}^+$, $p_{(q)}^-$ at the inlet and outlet from the corresponding hydraulic unit (valve, hydraulic cylinder, etc.)

$$p_{(q)} = p_{(q)}^+ , \quad p_{(q)}^+ = p_k^+ , \quad k \in \gamma_{(q)}^+ , \quad p_{(q)}^- = p_l^- , \quad l \in \gamma_{(q)}^- . \tag{15}$$

Using the equations of hydraulic-cylinder dynamics as an example, we consider approximation of mathematical models of the nodes of the hydraulic network. We approximate Eqs. (5), (6) to $\omega_{h\tau}$ by the difference scheme

$$f_1 \overset{m+1}{p_{(q)}^+} - f_2 \overset{m+1}{p_{(q)}^-} = m \frac{\overset{m+1}{u} - u}{\tau} + b \overset{m+1}{u} + F_1 (\overset{m}{z}) + \frac{d\overset{m}{F_1}}{dz} \tau (\overset{m+1}{u} - \overset{m}{u}) + F_2 (t_{j+1}) , \tag{16}$$

$$\frac{\overset{m+1}{p_{(q)}^+} - p_{(q)}^+}{\tau} k_1 = f_k \overset{m+1}{v}_{k,N_k} - f_1 \overset{m+1}{u} , \quad \frac{\overset{m+1}{p_{(q)}^-} - p_{(q)}^-}{\tau} k_2 = f_2 \overset{m+1}{u} - f_l \overset{m+1}{v}_{l,0} . \tag{17}$$

Rearranging (16), we obtain

$$\overset{m+1}{p_{(q)}^-} = \left(f_1 \overset{m+1}{p_{(q)}^+} - \mu_1 f_l \overset{m+1}{v}_{l,0} \Big/ f_2 + \mu_2 \right) \Big/ \mu_3 , \tag{18}$$

where

$$\mu_1 = \frac{m}{\tau} + b + \frac{d\overset{m}{F_1}}{dz} \tau , \quad \mu_2 = \frac{\mu_1 k_2 p_{(q)}^+}{\tau f_2} + \frac{mu}{\tau} - F_1 (\overset{m}{z}) + \frac{d\overset{m}{F_1}}{dz} \tau \overset{m}{u} - F_2 (t_{j+1}) , \quad \mu_3 = \frac{\mu_1 k_2}{\tau f_2} + f_2 .$$

$m+1$ Using (18) and relations (14), (15), we express in (13) $\overset{m+1}{p_k^-}$, $\overset{m+1}{p_k^+}$ in terms of the elements $\overset{m+1}{p}_{(q)}$ of the vector p of nodal pressures. The system of linear equations obtained is presented in the form

$$A_{\langle k\rangle}\overset{m+1}{\mathbf{r}_{\langle k\rangle}} - 2P_{\langle k\rangle}\overset{m+1}{\mathbf{p}} = \mathbf{b}_{\langle k\rangle}, \quad k\in\gamma, \tag{19}$$

$$\overset{m+1}{\mathbf{r}_{\langle k\rangle}} = \left(\overset{m+1}{r_{k,0}},\ \overset{m+1}{r_{k,N_k}}\right)', \quad \overset{m+1}{\mathbf{p}} = \left(\overset{m+1}{p_{(1)}},\ \overset{m+1}{p_{(2)}}, \ldots, \overset{m+1}{p_{(n)}}\right)',$$

$$A_{\langle k\rangle} = (a^{\langle k\rangle}_{\alpha\beta},\ \alpha,\beta = 1,2), \quad \left|a^{\langle k\rangle}_{11}\right| > \left|a^{\langle k\rangle}_{12}\right|, \quad \left|a^{\langle k\rangle}_{22}\right| > \left|a^{\langle k\rangle}_{21}\right|,$$

$$P_{\langle k\rangle} = (p^{\langle k\rangle}_{\alpha\beta},\ \alpha = 1,2,\ \beta = \overline{1,n}),$$

$$p^{\langle k\rangle}_{1\beta} = \begin{cases} 1, & k\in\overset{-}{\gamma}_{(\beta)}, \\ 0, & k\notin\overset{-}{\gamma}_{(\beta)}, \end{cases} \quad p^{\langle k\rangle}_{2\beta} = \begin{cases} 1, & k\in\overset{+}{\gamma}_{(\beta)}, \\ 0, & k\notin\overset{+}{\gamma}_{(\beta)}. \end{cases}$$

The difference analogue of fluid flow-rate equations (17) with allowance for (18) takes the form

$$\overset{+}{b}_{\langle k\rangle} f_k \overset{m+1}{v_{k,N_k}} + \overset{-}{b}_{\langle l\rangle} f_l \overset{m+1}{v_{l,0}} + u_{qq}\overset{m+1}{\overset{+}{p}_{(q)}} = d_q. \tag{20}$$

Here

$$\overset{+}{b}_{\langle k\rangle} = \frac{1}{f_1}, \quad \overset{-}{b}_{\langle l\rangle} = -\frac{1}{f_2}\left(1 - \frac{\mu_1 f_l k_2}{\mu_3 f_2 \tau}\right),$$

$$u_{qq} = -\frac{1}{\tau}\left(\frac{k_1}{f_1} + \frac{f_l k_2}{\mu_3 f_2}\right), \quad d_q = \frac{1}{\tau}\left(\frac{\mu_2 k_1}{\mu_3 f_2} - \frac{k_1 \overset{+}{p}_{(q)}}{f_1} - \frac{k_2 \overset{-}{p}_{(q)}}{f_2}\right).$$

Consequently, the balance equations in the inner nodes of the hydraulic network can be written in the form

$$\sum_{k\in\gamma} f_k B_{\langle k\rangle}\overset{m+1}{\mathbf{v}_{\langle k\rangle}} + U\overset{m+1}{\mathbf{p}} = \mathbf{d}, \tag{21}$$

$$\overset{m+1}{\mathbf{v}_{\langle k\rangle}} = \left(\overset{m+1}{v_{k,0}},\ \overset{m+1}{v_{k,N_k}}\right)', \quad U = (u_{\alpha\beta},\ \alpha,\beta = \overline{1,n},\ u_{\alpha\beta} = 0,\ \alpha\neq\beta), \quad \mathbf{d} = (d_1, d_2, \ldots d_n)',$$

$$B_{\langle k\rangle} = (b^{\langle k\rangle}_{\alpha\beta},\ \alpha = \overline{1,n},\ \beta = 1,2), \quad b^{\langle k\rangle}_{\alpha 1} = \begin{cases} \overset{-}{b}_{\langle k\rangle}, & k\in\overset{-}{\gamma}_{(\alpha)}, \\ 0, & k\notin\overset{-}{\gamma}_{(\alpha)}, \end{cases} \quad b^{\langle k\rangle}_{\alpha 2} = \begin{cases} \overset{+}{b}_{\langle k\rangle}, & k\in\overset{+}{\gamma}_{(\alpha)}, \\ 0, & k\notin\overset{+}{\gamma}_{(\alpha)}. \end{cases}$$

Since the representation

$$\overset{m+1}{\mathbf{v}_{\langle k\rangle}} = \left(C_{\langle k\rangle}\overset{m+1}{\mathbf{r}_{\langle k\rangle}} + \mathbf{g}_{\langle k\rangle}\right)\Big/(2\rho c_k), \quad C_{\langle k\rangle} = \begin{pmatrix} 1 & -\overset{-}{\xi}_k \\ \overset{+}{\xi}_k & -1 \end{pmatrix}, \quad \mathbf{g}_{\langle k\rangle} = (-\overset{-}{v}_k,\ \overset{+}{v}_k)',$$

takes place for the vector $\overset{m+1}{\mathbf{v}_{\langle k\rangle}}$, balance equations (21) in the Riemann invariants can be written as

$$\sum_{k\in\gamma} f_k B_{\langle k\rangle} C_{\langle k\rangle}\overset{m+1}{\mathbf{r}_{\langle k\rangle}}/c_k + 2\rho U\overset{m+1}{\mathbf{p}} = 2\rho\,\mathbf{d} - \sum_{k\in\gamma} f_k B_{\langle k\rangle}\overset{m+1}{\mathbf{g}_{\langle k\rangle}}/c_k. \tag{22}$$

Relations (19), (22) are the system of linear equations relative to $2M + n$ unknowns. We note that det $A_{\langle k\rangle}\neq 0$. Consequently

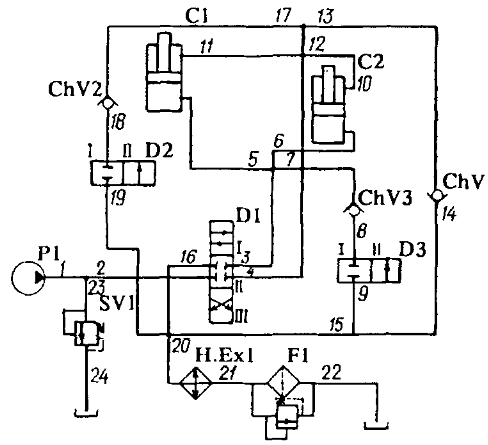


Fig. 1. Schematic of the hydraulic drive of the mechanism for lowering cargo (1-24, pipeline numbers).

$$\overset{m+1}{\mathbf{r}}_{\langle k\rangle} = 2A^{-1}_{\langle k\rangle} P_k \overset{m+1}{\mathbf{p}} + A^{-1}_{\langle k\rangle} \mathbf{b}_{\langle k\rangle}, \quad k \in \gamma . \tag{23}$$

Substituting (23) into (22), we obtain the system of n linear equations

$$Z \overset{m+1}{\mathbf{p}} = \rho\, \mathbf{d} - 0.5 \sum_{k\in\gamma} f_k B_{\langle k\rangle} (C_{\langle k\rangle} A^{-1}_{\langle k\rangle} \mathbf{b}_{\langle k\rangle} + \mathbf{g}_{\langle k\rangle})/c_k . \tag{24}$$

The elements of the matrix $Z = \sum f_k B_{\langle k\rangle} C_{\langle k\rangle} A^{-1}_{\langle k\rangle} P_{\langle k\rangle}/c_k + \rho U$ of the system (24) are determined by the formulas

$$z_{\alpha\beta} = \sum_{k\in\gamma^-_{(\alpha)}\cap\gamma^+_{(\beta)}} b^{\langle k\rangle}_{\alpha 1} (a^{\langle k\rangle}_{12} + \xi^-_k a^{\langle k\rangle}_{11}) \phi^{\langle k\rangle} + \sum_{k\in\gamma^+_{(\alpha)}\cap\gamma^-_{(\beta)}} b^{\langle k\rangle}_{\alpha 2} (a^{\langle k\rangle}_{21} + \xi^+_k a^{\langle k\rangle}_{22}) \phi^{\langle k\rangle}, \quad \alpha \neq \beta ,$$

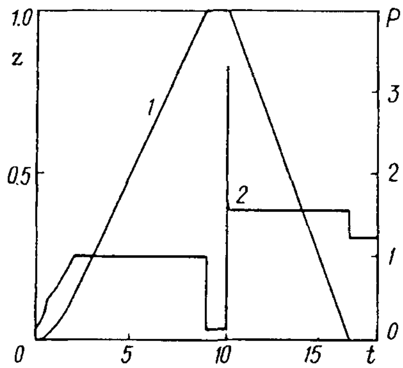
$$a_{\alpha\alpha} = \sum_{k\in\gamma^-_{(\alpha)}} b^{\langle k\rangle}_{\alpha 1} (a^{\langle k\rangle}_{22} + \xi^-_k a^{\langle k\rangle}_{21}) \phi^{\langle k\rangle} - \sum_{k\in\gamma^+_{(\alpha)}} b^{\langle k\rangle}_{\alpha 2} (a^{\langle k\rangle}_{11} + \xi^+_k a^{\langle k\rangle}_{12}) \phi^{\langle k\rangle} + \rho u_{\alpha\alpha} ,$$

$$\phi^{\langle k\rangle} = f_k/(c_k (a^{\langle k\rangle}_{11} a^{\langle k\rangle}_{22} - a^{\langle k\rangle}_{12} a^{\langle k\rangle}_{21})) .$$

Thus, the original initial-boundary-value problem for the equations of a weakly compressible fluid in Riemann invariants is reduced by the Newton-type iteration process to the solution of linear algebraic equations (24) relative to n unknowns $\overset{m+1}{p}_{(1)}, \overset{m+1}{p}_{(2)}, \ldots, \overset{m+1}{p}_{(n)}$ in the graph nodes. Then from (23) we find the invariants $\overset{m+1}{r}_{k,0}$, $\overset{m+1}{s}_{k,N_k}$ at the boundaries of the graph edges. The values of the invariants in the remaining nodes of the grid are determined by the equations of "running calculation" (11).

3. **Computational Experiment.** The algorithms constructed were used to study unsteady fluid flows in the hydraulic drive of a mechanism for lowering cargo (Fig. 1). The scheme of the hydraulic drive includes the pump P1, force cylinders C1, C2, distributors D1, D2, D3, the safety valve SV1, check valves ChV1, ChV2, ChV3, the heat exchanger H.Ex1, the filter for cleaning the working fluid F1, and the necessary connecting pipelines.

The lengths of the pipelines varied from 0.1 to 1.58 m and the diameters from 0.025 to 0.048 m. The thickness of the pipeline walls δ_k was $2.5 \cdot 10^{-3}$ m, their roughness was $\Delta_k = 5 \cdot 10^{-5}$ m, the elasticity modulus of the material $E_w = 2.06 \cdot 10^{-11}$ N/m^2. The coefficient of hydraulic drag λ_k was found by the formula



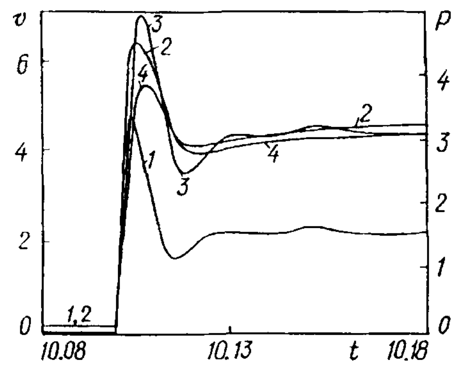


Fig. 2. Displacement $z(t)$ of pistons of cylinders C1, C2 (1) and change in pressure in pipeline 20 (2). z, m; p, MPa; t, sec.

Fig. 3. Change in pressure [$n_{\text{h.ex}}$ = 10 (1), 25 (2)] and velocity [$n_{\text{h.ex}}$ = 10 (3), 25 (4)] in pipeline 20. v, m/sec.

$$\lambda_k = \begin{cases} 64/\mathrm{Re}_k\,, & \mathrm{Re}_k \le 2320\,, \\ 0.11\,(\Delta_k/d_k + 68/\mathrm{Re}_k)^{0.25}\,, & \mathrm{Re}_k > 2320\,. \end{cases}$$

The safety valve SV1 is adjusted to pressure $\Delta p = 2.0 \cdot 10^7$ Pa, and the pressure of adjustment of check valves is $5 \cdot 10^5$ Pa. Cylinders C1, C2 and the heat exchanger H.Ex1 had the following characteristics: $D_{\text{cyl}} = 0.18$ m, $D_r = 0.08$ m, $L_{\text{cyl}} = 1$ m, $m = 3500$ kg, $b = 0$ kg/sec, $F(z) = 0$ N, $F(t) = 3.5 \cdot 10^4$ N, $n_{\text{h.ex}} = 10$, $l_{\text{h.ex}} = 1$ m, $d_{\text{h.ex}} = 0.04$. Fluid feed by the pump P1 was $Q_p = 190t$ liters/min, $t \le 2$ sec, $Q_p = 380$ liters/min, $t > 2$ sec. Working fluid parameters were $\rho = 840$ kg/m^3, $\nu = 2 \cdot 10^{-5}$ m^2/sec, $F_{\text{fl}} = 1.6 \cdot 10^9$ N/m.

Distributors D1-D3 perform the control functions over cylinders C1, C2 in the hydraulic system. Cargo is lifted by switching the distributor D1 to the position III. The time of lifting cargo of weight $7 \cdot 10^3$ kg was 9 sec (Fig. 2). Switching of D1 to the position I, and D2 and D3 to the position II (mode of cargo lowering) took place at the time instant $t = 10.1$ sec. In this case, a short-term increase in pressure up to 3.29 MPa was observed in the drainage hydraulic line (pipeline 20). Change in pressure and fluid flow rate in the drainage hydraulic line as a function of passes $n_{\text{h.ex}}$ of the heat exchanger H.Ex1 is shown in Fig. 3. The time of cargo lowering was 6.4 sec. The deviation of the results of the computational experiment from the data obtained during a full-scale experiment did not exceed 9%.

The calculation results presented are obtained by the program complex of the automated computational experiment developed by the authors for simulating processes in hydraulic networks. The complex of programs involves the dialog monitor; graphical editor, which allows the creation and correction of principal circuits of hydraulic systems; special program aids for solving nonlinear equations of hydrodynamics in hydraulic networks; and subsystem for analyzing and processing simulation results. The program complex is intended for studying processes in systems involving the following hydraulic units: pipelines, pumps, safety and check valves, hydraulic engines, pneumatic-hydraulic storage batteries, hydraulic cylinders, distributors, and filters.

Conclusions. Computational algorithms are developed to solve nonlinear equations of hydraulics in hydraulic networks. The algorithms suggested can be used to study unsteady processes in hydraulic drives during their design.

NOTATION

p, v, ρ, ν, pressure, velocity, density, and kinematic viscosity of fluid; d_k, f_k, δ_k, Δ_k, diameter, cross-section area, thickness, and roughness of walls of the kth pipeline; $\mathrm{Re}_k = v_k d_k/\nu$, Reynolds number; E_{fl}, E_{w}, bulk elasticity moduli of fluid and wall material; E_{red}, reduced elasticity modulus; c_k, velocity of sound propagation in the kth pipeline; $p_{(q)}$, fluid pressure in the node q of the hydraulic network; b, coefficient of viscous friction; z, u, L_{cyl}, displacement, velocity, and path of the piston; m, mass of movable parts of the hydraulic cylinder reduced to the rod; $f_1 = 0.25\pi D^2_{\text{cyl}}$; $f_2 = 0.25\pi(D^2_{\text{cyl}} - D^2_{\text{r}})$, working areas of pistons in cavities 1 and 2; D_{cycl}, D_{r}, diameters of cylinder and rod; $p^{+}_{(q)}$, $p^{-}_{(q)}$, fluid pressures in cavities 1 and 2; $F_1(z)$, $F_2(z)$, loads on the rod; k_1, k_2, coefficients of elasticity of hydraulic-cylinder cavities; Δp_{v}, pressure of valve adjustment.

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